# The University of British Columbia
## Department of Mathematics
## Qualifying Examination—Analysis
January 9, 2016

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1. (10 points) Let $\vec{F} = (x^2 + y^2 + z^2)\,\hat{\boldsymbol{\imath}} + (e^{x^2} + y^2)\,\hat{\boldsymbol{\jmath}} + (3 + x + z^2)\,\hat{\mathbf{k}}$ and let $S$ be the part of the surface $x^2 + y^2 + z^2 = 2az + 3a^2$ having $z \geq 0$, oriented with the normal pointing away from the origin. Compute the flux, $\iint_S \vec{F} \cdot \hat{\mathbf{n}}\, dS$, of $\vec{F}$ through $S$.

2. (10 points) Prove that a real-valued function $f$ is uniformly continuous on $(0, 1)$ if and only if it extends to a real-valued continuous function on $[0, 1]$.

3. (10 points) Let $a_{ij}$ be a real number for every $i, j \geq 1$. Suppose

   (i) the series $\sum_{i=1}^{\infty} a_{ij}$ converges absolutely to a real number $S_j$ for every $j \geq 1$,

   (ii) the series $\sum_{j=1}^{\infty} a_{ij}$ converges absolutely to a real number $T_i$ for every $i \geq 1$,

   (iii) the series $\sum_{j=1}^{\infty} S_j$ and $\sum_{i=1}^{\infty} T_i$ both converge absolutely.

   Can we conclude that $\sum_{j=1}^{\infty} S_j = \sum_{i=1}^{\infty} T_i$? Give a proof or a counterexample.

4. (10 points) Evaluate the integral $\displaystyle\int_0^{\infty} \frac{x^{-3/4}}{x+1}\, dx$.

5. (10 points) Let $f(z) = \displaystyle\sum_{n=0}^{\infty} a_n z^n$ be analytic in a domain containing $\{z : |z| \leq 1\}$.

   Give a careful proof, justifying all steps, that

$$\frac{1}{2\pi} \int_0^{2\pi} |f(e^{i\theta})|^2 d\theta = \sum_{n=0}^{\infty} |a_n|^2.$$

6. (10 points) Find all one-to-one entire functions of a complex variable $z$.