Be sure that this examination has 11 pages including this cover

The University of British Columbia

Sessional Examinations - December 2006

Mathematics 100/180

Differential Calculus with Applications to Physical Sciences and Engineering

Closed book examination

Time: 2.5 hours

Last Name:	First Name:
Student Number:	Instructor's Name:
Signature:	Section Number:

Rules governing examinations

1. Each candidate should be prepared to produce his or her library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1	42
2	10
3	12
4	16
5	10
6	10
Total	100

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Marks

[42] 1. Short-Answer Questions. Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a) Evaluate
$$\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 16}$$

Answer

(b) Evaluate
$$\lim_{x \to \infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4}$$

Answer

(c) Find the value of the constant *a* for which
$$\lim_{x \to -2} \frac{x^2 + ax + 3}{x^2 + x - 2}$$
 exists.

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(d) Find the values of the constants a and b for which

$$f(x) = \begin{cases} x^2 \text{ if } x \leq 2\\ ax + b \text{ if } x > 2 \end{cases}$$

is differentiable everywhere.

Answer

(e) Find the derivative of $e^{x \cos x}$.

Answer

(f) If $x^2y^2 + x \sin y = 4$, find dy/dx.

Answer

(g) Let $f(x) = x^3 \ln x$. Find the 4th derivative of f(x), i.e. $f^{(4)}(x)$.

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(h) Find the derivative of $\sin^{-1}(e^{2x})$. Note: Another notation for \sin^{-1} is arcsin.

Answer		

(i) Find the derivative of $(\tan x)^x$.

Answer

(j) Using a suitable linear approximation, estimate $(8.06)^{2/3}$. Give your answer as a fraction in which both the numerator and denominator are integers.

Answer

(k) Write down the first three nonzero terms in the Maclaurin series for $x \sin(-2x)$.

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(1) A function f(x) has third derivative equal to 10/(1-x). The second-degree Taylor polynomial $T_2(x)$ at 0 is used to approximate f(0.1). Find the upper bound for the error given by Taylor's Inequality, i.e. the upper bound for $|f(0.1) - T_2(0.1)|$.

Answer

(m) Using Newton's Method, starting with $x_1 = 1$, find the approximation x_3 to the root of the equation $x^3 - x^2 - 1 = 0$.

Answer

(n) A particle moves in a straight line so that its velocity at time t is $v(t) = t^3$. If its position at time t is s(t) and s(1) = 2, find s(2).

Answer	

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Full-Solution Problems. In questions 2–6, justify your answers and **show all your work**. If a box is provided, write your final answer there. Simplification of answers is not required.

[10] 2. A poster is to have a total area of 180 cm² with 1-cm margins at the bottom and sides and a 2-cm margin at the top. What dimensions will give the largest printed area?

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- [12] 3. Two cities, Growth and Decay, have populations that are respectively increasing and decreasing, at (different) rates that are proportional to the respective current population. Growth's population is now 3 million and was 2 million 10 years ago, and Decay's population is now 5 million and was 7 million 10 years ago.
 - (a) (6 marks) How many years from now will Growth's population equal 4 million?

Answer

(b) (6 marks) How many years from now will Growth and Decay have the same population?

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[16] **4.** The function f(x) is defined by

$$f(x) = \begin{cases} e^x & \text{if } x < 0\\ \frac{x^2 + 3}{3(x+1)} & \text{if } x \ge 0 \end{cases}$$

(a) (2 marks) Explain why f(x) is continuous everywhere.

- (b) Determine all of the following if they are present:
- (i) (4 marks) x-coordinates of local maxima and minima, intervals where f(x) is increasing or decreasing;

(ii) (4 marks) intervals where f(x) is concave upwards or downwards;

Question 4(b) continued on the next page...

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Question 4(b) continued

(iii) (3 marks) equations of any asymptotes (horizontal, vertical, or slant).

(c) (3 marks) Sketch the graph of y = f(x), giving the (x, y) coordinates for all points of interest above.

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[10] 5. A trough is 10 m long and its ends have the shape of equilateral triangles (i.e. all three sides have equal length) that are 2 m across, as shown in the diagram below. If the trough is being filled with water at the rate of 12 m³/min, how fast is the water level rising when the water is 60 cm deep?



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[10] 6. (a) (6 marks) There are two distinct straight lines that pass through the point (1, -3) and are tangent to the curve $y = x^2$. Find equations for these two lines.

Answer

(b) (4 marks) Evaluate

$$\lim_{x \to 0} x^{1/101} \sin(x^{-100})$$

or explain why this limit does not exist. Give complete justification for your answer.