The University of British Columbia

Final Examination - December 10, 2013

Mathematics 100/180

All Sections

Closed book examination Time: 2.5 hours

Last Name ____________ **First** ________ **Signature** ____________

MATH 100 or MATH 180 (Circle one) **Student Number** ____________

Instructor's Name ____________ **Section Number** ____________

Special Instructions:

No books, notes, or calculators are allowed.

Student Conduct during Examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  (a) speaking or communicating with other candidates, unless otherwise authorized;
  (b) purposely exposing written papers to the view of other candidates or imaging devices;
  (c) purposely viewing the written papers of other candidates;
  (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		12
2		9
3		9
4		9
5		7
6		7
7		7
8		17
9		8
10		10
11		5
Total		100

[12] **1. Short-Answer Questions**. Questions 1–4 are short-answer questions. Put your answers in the boxes provided. **Simplify your answers as much as possible, and show your work.** Each question is worth 3 marks, but not all questions are of equal difficulty.

(a) Evaluate the limit $\lim_{x \to -5} \dfrac{x^2 + 2x - 15}{2x + 10}$.

Answer:

(b) If $2x - 3 \le f(x) \le x^2 - 4x + 6$ for $x \ge -1$, find $\lim_{x \to 3} f(x)$.

Answer:

(c) Evaluate the limit $\lim_{x \to 0} \frac{x}{\sqrt{1+2x}-1}$.

Answer:

(d) A bacteria culture initially contains 50 cells and grows at a rate proportional to its size. After an hour the population has increased to 250. Find an expression for the number of bacteria after t hours.

Answer:

[9] **2.**

(a) Find an equation of the tangent line to the curve $y = e^{\sin x} + x$ at the point $(0, 1)$.

Answer:

(b) If $y = (\ln x)^x$, find $\dfrac{dy}{dx}$.

Answer:

(c) Suppose that $f(1) = -1$, $g(-1) = 2$, $f'(1) = -2$, and $g'(-1) = 3$. Find $h'(-1)$, where $h(x) = f(x^2)g(x)$.

Answer:

[9] **3.**

(a) If a function $y = f(x)$ is defined implicitly by an equation $x^3 + xy = 5x - y^3$, find $\dfrac{dy}{dx}$.

Answer:

(b) Let $s = f(t) = t^3 - \dfrac{21}{2}t^2 + 30t$ be the position function of a particle that is moving in a straight line, where t is measured in seconds and s in metres. When is the particle moving in the negative direction?

Answer:

(c) Let $f(x) = \begin{cases} cx^2 + 3 & \text{if } x \geq 1 \\ 2x^3 - c & \text{if } x < 1 \end{cases}$, where c is a constant. For what value of the constant c is the function $f(x)$ continuous on $(-\infty, \infty)$?

Answer:

[9] **4.**

(a) Use Newton's method with an initial approximation of $x_1 = -1$ to find x_2, where x_2 is the second approximation to the root of the equation $x^7 + 4 = 0$.

Answer:

(b) The graph of $y = f(x) = \dfrac{x^3 + x^2}{x^2 + 1}$ has a slant asymptote. Find an equation of the slant asymptote.

Answer:

(c) Let $f(x) = 4x^3$. What is the 2nd degree Taylor polynomial $T_2(x)$ centered at $a = 1$ for $f(x)$?

Answer:

Full-Solution Problems. In questions 5–12, justify your answers and **show all your work**. If a box is provided, write your final answer there. **Unless otherwise indicated, simplification of numerical answers is required in these questions**.

[7] **5**. A bottle of soda pop at room temperature (70°F) is placed in a refrigerator where the temperature is 40°F. After half an hour the soda pop has cooled to 60°F. How long does it take for the soda pop to cool to 50°F?

[7] **6**. Using the definition of the derivative, compute $f'(x)$ if $f(x) = \dfrac{x}{2x+1}$. No marks will be given for the use of differential rules, but you may use them to check your answer.

[7] **7**. At a distance of 4 kilometres from the launch site, a spectator is observing a rocket being launched. If the rocket lifts off vertically and is rising at a speed of 0.7 kilometres/second when it is at an altitude of 3 kilometres, how fast is the distance between the rocket and the spectator changing at that instant?

[17] **8**. Let $y = f(x) = 63x^{2/7} - 14x^{9/7}$.

(a) Find the critical numbers of $y = f(x)$.

Answer:

(b) Determine the intervals where $f(x)$ is increasing, and the intervals where $f(x)$ is decreasing.

Answer:

(c) Determine the intervals where $f(x)$ is concave up, and the intervals where $f(x)$ is concave down.

Answer:

(d) Sketch the graph of $y = f(x)$ and indicate the inflection point(s) on your graph.

[8] **9**. Find the area of the largest rectangle which has two vertices on the x-axis and two vertices lie on the graph of the function $y = 8 - x^2$ with $-\sqrt{8} \le x \le \sqrt{8}$. Please justify your answer.

[10] **10.**

(a) Find the first Maclaurin polynomial for $f(x) = \ln(1 - x^2)$.

Answer:

(b) Using the Lagrange Remainder Formula for $f(x) = \ln(1 - x^2)$ prove that $-\dfrac{5}{32} \leq \ln\left(\dfrac{8}{9}\right) \leq -\dfrac{1}{9}$.

Extra space provided to answer 10.(b)

[5] **11**. Prove that there exists at most one positive real number x such that $4^x = 3\cos x$.

The End