The University of British Columbia

Final Examination - December 8, 2009

Mathematics 104/184

All Sections

Closed book examination   Time: 2.5 hours

Last Name _____ **First** _____ **Signature** _____

Student Number _____

MATH 104 or MATH 184 (Circle one)   **Section Number:**_____

Special Instructions:

No memory aids are allowed. One Sharp EL-510R calculator, WITH COVER REMOVED, may be used. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

1		42
2		14
3		12
4		12
5		12
6		8
Total		100

[42] **1**. **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Find a such that $\displaystyle\lim_{x\to\infty} \frac{4x^2 + x + a}{ax^2 - 7x} = 2$.

Answer:

(b) Evaluate $\displaystyle\lim_{x\to 4} \frac{\sqrt{2x-1} - \sqrt{7}}{x-4}$.

Answer:

(c) Find all numbers c that make

$$f(x) = \begin{cases} \dfrac{x-c}{c+1} & \text{if } x \le 0 \\[2mm] x^2 + c & \text{if } x > 0 \end{cases}$$

continuous for every x in its domain.

Answer:

(d) If the quantity q sold per week of a product is related to its unit price p by $q = 7 - 2\ln p$, find the *marginal revenue* as a function of price. SIMPLIFY your answer.

Answer:

(e) Give the coordinates (x, y) of the local maximum of $y = \dfrac{x}{1 + x^2}$.

Answer:

(f) Give the x-coordinate(s) of the absolute minimum of $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$ on the interval $[0, 2]$.

Answer:

(g) Find the y-intercept of the tangent line to the curve $y^2 = x^3 + 1$ at the point $(2, -3)$.

> Answer:

(h) Let $f(x)$ be a function with the property that $f'(x) = \dfrac{x-1}{x+1}$. If $g(x) = f(e^x)$, compute $g'(x)$.

> Answer:

(i) Compute the absolute value of the difference between the calculator value of $\sqrt[3]{28}$ and the approximate value of $\sqrt[3]{28}$ computed using the linear approximation to $f(x) = \sqrt[3]{x}$ at $x = 27$.

> Answer:

(j) The demand equation relating the price p and quantity demanded q for a particular luxury good is $e^{pq} - q^2 = 1$. Find the rate, dp/dt, at which the price p is changing if the revenue is increasing at $1/e$ dollars per day when $p = \ln(2)$ and $q = 1$.

Answer:

(k) Investment A is worth \$70,200 and is growing at a rate of 13% per year compounded continuously. Investment B is currently worth \$60,000 and is growing at a rate of 14% per year compounded continuously. After how many years will the two investments have the same value?

Answer:

(l) Find $f'(x)$ where $f(x) = 2^{\sin(x)}$.

Answer:

(m) Find the interval(s) on which $f(x) = \dfrac{9}{14}x^{1/3}(x^2 - 7)$ is concave up.

Answer:

(n) Determine the *third* degree Taylor polynomial of $f(x) = \cos(\pi - 5x)$ at $x = 0$.

Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[14] **2**. Let $y = f(x) = \dfrac{(x+1)^2}{x^2+1}$.

(a) Find the intervals on which $f(x)$ is increasing and on which it is decreasing. [4pts]

(b) Find the intervals on which $f(x)$ is concave upward and on which it is concave downward, as well as the x-coordinates of any inflection points. [4pts]

(c) Find any asymptotes for $y = f(x)$. [2pts]

(d) Sketch the graph of $y = f(x)$. Identify on your graph the coordinates of any local maxima and local minima, and of any inflection points. Also, indicate any asymptotes that exist. [4pts]
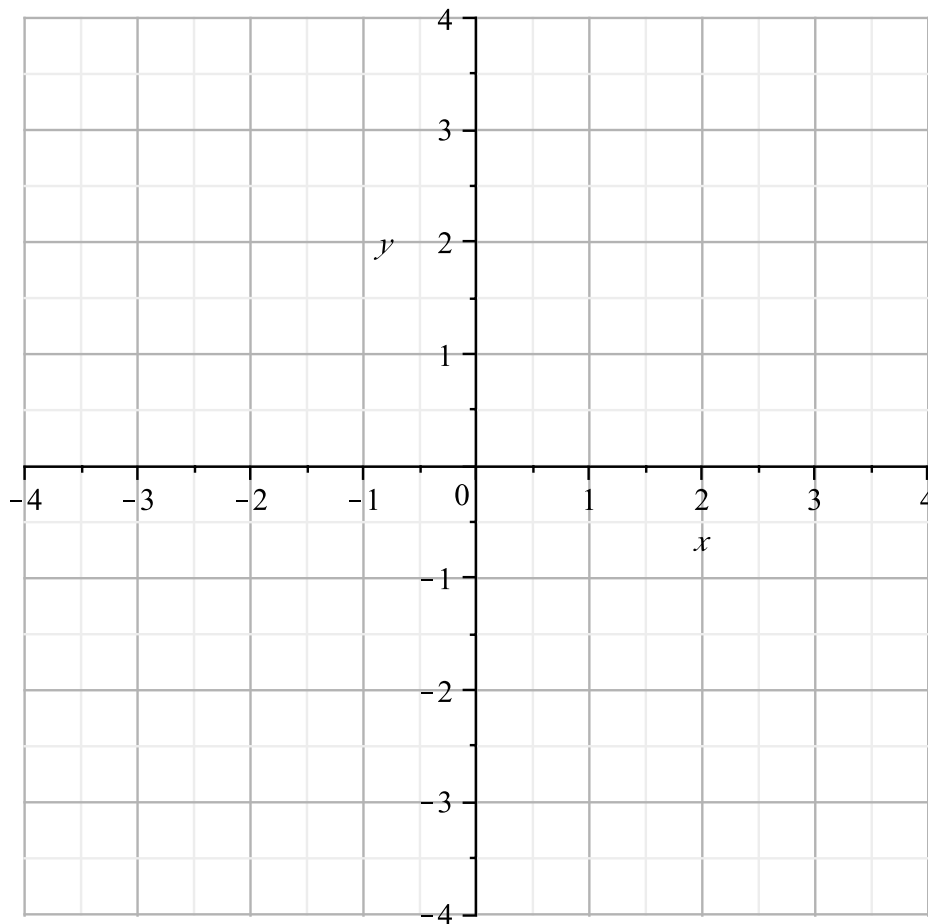
[12] **3**. At the moment a cyclist passes directly beneath a balloon, the balloon is 40 metres above the ground. The cyclist is travelling along a straight road at a constant speed of 10 metres per second, and the balloon is rising at a constant rate of 4 metres per second. How fast is the distance between the cyclist and the balloon changing 5 seconds after the balloon is directly above the cyclist?

Answer:

[12] **4.** Shark Inc. has determined that demand for its newest netbook model is given by

$$\ln q - 2\ln p + 0.02p = 7,$$

where q is the number of netbooks Shark can sell at a price of p dollars per unit. Shark has determined that this model is valid for prices $p \geq 100$. You may find it useful in this problem to know that elasticity of demand is defined to be $E(p) = -pf'(p)/f(p)$ for the demand function $q = f(p)$.

(a) If the current price is \$200 per unit, will revenue increase or decrease if the price is lowered slightly?

Answer:

(b) Find the price that maximizes the revenue from sales of this netbook model. Be sure to justify that you have found the maximum.

[12] **5**. GoFast Motorcycles is the sole agent licensed to sell the Hinda 250-cc motorcycle in British Columbia. Management estimates that the demand for these motorcycles is 10 000 per year, and that they will sell at a uniform rate throughout the year. The cost incurred on ordering each shipment of motorcycles is $10 000, and the cost of storing each motorcycle is $200 per year. How often should GoFast Motorcycles order motorcycles to minimize the inventory costs? Note that you should compute the carrying costs based on the average number of motorcycles stored during the order-reorder period.

Answer:

[8] **6**. A loan of $100,050 is repaid over 20 years by monthly payments of $900. Use two iterations of the Newton-Raphson method starting with the initial guess $i_0 = 0.02$ to determine the monthly rate of interest. (Note that if P is borrowed at a rate of i per period, and is repaid over N periods by payments of R, then $Pi + R[(1+i)^{-N} - 1] = 0$.)

Answer:

The End