Name (print):

Student number:

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**University of British Columbia**

**MATH 110: APRIL EXAM**

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Date: *April 24, 2014*

Time: *3:30 p.m. to 6:00 p.m.*

Number of pages: *13 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

*Each candidate must be prepared to produce, upon request, a UBC card for identification.*

*No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.*

*Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:*

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

*Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.*

*Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.*

| For examiners' use only | | |
|---|---|---|
| Question | Mark | Possible marks |
| 1 | | 8 |
| 2 | | 7 |
| 3 | | 17 |
| 4 | | 6 |
| 5 | | 7 |
| 6 | | 6 |
| 7 | | 5 |
| 8 | | 4 |
| 9 | | 5 |
| Total | | 65 |

You do not have to simplify your answers. “Calculator-ready” answers are sufficient.

1. Determine whether each of the following statements is true or false. If it is true, provide justification. If it is false, provide a counterexample.

   (a) **[2 marks]** If $f$ is defined on the closed interval $[-1, 1]$, then $f$ attains a global maximum on that interval.

   (b) **[2 marks]** If $f$ is differentiable everywhere and $f(x) = 0$ has two solutions, then $f'(x) = 0$ has at least one solution.

   (c) **[2 marks]** $f(x) = \sin x$ has infinitely many inflection points.

   (d) **[2 marks]** If $f(1) = 2$, then $f$ does not have a vertical asymptote at $x = 1$.

2. For each of the following short answer questions, underline your final answer.

(a) **[2 marks]** Find $\lim_{x \to 2} \frac{x^4 - 8x}{x^2 - 4}$.

(b) **[3 marks]** Find $\lim_{x \to \infty} \frac{5x^2 + \ln x}{2x^2 + 3x}$.

(c) **[2 marks]** State either the Mean Value Theorem or Rolle's Theorem.

3. Let $f(x) = \dfrac{x^2}{(x-4)^2}$.

(a) **[1 mark]** State the domain of $f$.

(b) **[3 marks]** Find all the vertical and horizontal asymptotes of $f$, if there are any.

(c) **[2 marks]** Compute $f'(x)$.

(d) **[3 marks]** Find the intervals where $f$ is increasing and the intervals where it is decreasing.

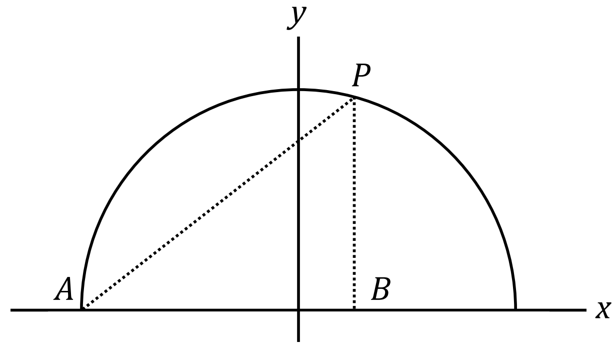
(e) **[2 marks]** Compute $f''(x)$.

(f) **[3 marks]** Find the intervals where $f$ is concave up and the intervals where it is concave down.

(g) **[3 marks]** Make a large sketch of the graph of the function.

4. [**6 marks**] A farmer wants to enclose a rectangular pasture of 500 square metres. Three sides of the pasture will be enclosed with cedar fencing at a cost of \$10 per metre. The remaining side is to be enclosed with a stone wall at a cost of \$15 per metre. What should the dimensions of the field be to minimize the cost of the enclosure?

5. [**7 marks**] Find the coordinates of the point $P$ on the semicircle $y = \sqrt{1 - x^2}$ of radius 1 (pictured below) for which the right triangle $ABP$ has maximal area.



6. [**6 marks**] At noon, ship $A$ is 30 kilometres west of ship $B$. Ship $A$ is sailing east at 20 kilometres per hour and ship $B$ is sailing north at 30 kilometres per hour. How fast is the distance between the ships changing at 4:00 p.m. on the same day? (You should indicate in your answer if the distance between the ships is increasing or decreasing.)

7. A particle moves across the $x$-$y$ plane following the equation $10x = e^{2y} - e^{-2y}$.

   (a) **[1 mark]** Verify algebraically that the particle crosses the point $(0, 0)$.

   (b) **[4 marks]** Suppose we know that when the particle is at the point $(0, 0)$, the rate of change of its $x$-coordinate is $4$ metres per second. What is the rate of change of its $y$-coordinate at that point?

8. Imagine bushbuck antelopes interacting in a montane forest. Let $P$ denote the probability that a given antelope is in contact with another antelope. $P$ is modelled by the equation

$$P = 1 - e^{-\pi \rho D^2},$$

where $D$ is a constant denoting the “spotting distance” of the species, and $\rho$ is a function denoting the density of antelope in the area.

(a) **[2 marks]** Suppose $\rho'(t)$ is determined by field observations. Come up with an expression for the rate of change of the probability that an individual is in contact with another individual at time $t$. (Your answer will involve the term $\rho'(t)$.)

(b) **[2 marks]** Now suppose $\rho'(t) = 0$ for a period of time from $t = a$ to $t = b$. Make a large sketch of the graph of $P$ during that period.

9. (a) **[3 marks]** Use a linear approximation to estimate $\ln(0.9)$.

(b) **[1 mark]** Is your answer in part (a) an overestimate, an underestimate, or exactly equal to, the actual value of $\ln(0.9)$? Justify your answer.

(c) **[1 mark]** In some cases, the linear approximation of a function is equal to the function itself for all values of $x$. What is one such function?

This page may be used for rough work. It will not be marked.