Math 120 Final Exam December 2007 2.5 hours.

There are 12 pages in this test including this cover page. No calculators, books, notes, or electronic devices of any kind are permitted. Unless otherwise indicated, show all your work.

Rules governing formal examinations:

- 1. Each candidate must be prepared to produce his/her library/AMS card upon request;
- 2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions;
- 3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination;
- 4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action;
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
 - (b) Speaking or communicating with other candidates;
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received;
- 5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator; and
- 6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Problem #	Value	Grade
1	42	
2	16	
3	12	
4	6	
5	6	
6	8	
7	10	
Total	100	

I have read and understood the instructions and agree to abide by them.

Signed: _____

- 1. ([42 marks]) Short-Answer Questions. Put your answer in the box provided but show your work also. Each question is worth 3 marks. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.
 - (a) Evaluate $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$.

(b) Evaluate f'(2) if $f(x) = \ln(g(xh(x))), h(2) = 2, h'(2) = 3, g(4) = 3, g'(4) = 5.$

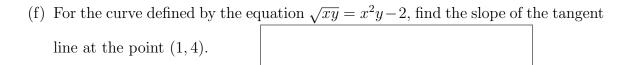
(c) Evaluate $\lim_{x\to-\infty} \frac{1-x-x^2}{2x^2-7}$.

(d) Find the values of the constants a and b for which

$$f(x) = \begin{cases} \cos(x) & x \le 0\\ ax + b & x > 0 \end{cases}$$

is differentiable everywhere.

(e) Find the derivative of
$$e^{\cos(x^2)}$$
.



(g) If $f(x) = \sin(x^2)$, compute $f^{(6)}(0)$. *Hint*: it may help to use Maclaurin polynomials.

(h) Find the (x, y) coordinates of all points where the graph of the parametric curve $x = \sin(t^2), y = \cos(t^2)$ has a vertical tangent.

(i) If $f(x) = (\cos x)^{\sin x}$, find f'(x).

- (j) Use a linear approximation to estimate $(2.001)^3$. Write your answer in the form
 - n/1000 where n is an integer.

(k) $f(x) = 2x - \sin(x)$ is one-to-one. Find $(f^{-1})'(\pi - 1)$.

(l) A point is moving on the unit circle { $(x, y) | x^2 + y^2 = 1$ } in the *xy*-plane. At $(2/\sqrt{5}, 1/\sqrt{5})$, its *y*-coordinate is increasing at rate 3. What is the rate of change

of its *x*-coordinate?

(m) Find the function y(t) if $\frac{dy}{dt} + 3y = 0$, y(1) = 2.

(n) For the function

$$f(x) = \begin{cases} 0 & x \le 0\\ \frac{\cos(x) - 1}{\sqrt{x}} & x > 0 \end{cases}$$

write in the box the (roman) number of the correct statement from the list:

- i. f is undefined at x = 0
- ii. f is neither continuous nor differentiable at x = 0
- iii. f is continuous but not differentiable at x = 0
- iv. f is differentiable but not continuous at x = 0
- v. f is both continuous and differentiable at x = 0

Full-Solution Problems. In questions 2-7, justify your answers and show all your work.

- 2. ([16 marks]) Let $f(x) = x\sqrt{3-x}$.
 - (a) ([2 marks]) Find the domain of f(x).

Answer

(b) ([4 marks]) Determine the x-coordinates of the local maxima and minima (if any) and intervals where f(x) is increasing or decreasing.

(c) ([2 marks]) Determine intervals where f(x) is concave upwards or downwards, and the x coordinates of inflection points (if any). You may use, without verifying it, the formula $f''(x) = (3x - 12)(3 - x)^{-3/2}/4$.

 \cdots question continued on next page

(d) ([2 marks]) There is a point at which the tangent line to the curve y = f(x) is vertical. Find this point.

(e) ([2 marks]) The graph of y = f(x) has no asymptotes. However, there is a real number *a* for which $\lim_{x\to-\infty} \frac{f(x)}{|x|^a} = -1$. Find the value of *a*.

Answer		

(f) ([4 marks]) Sketch the graph y = f(x), showing the features given in items (a) to (d) above and giving the (x, y) coordinates for all points occuring above.

3. ([12 marks]) What is the largest possible area of a window, with perimeter P, in the shape of a rectangle with a semicircle on top (so the diameter of the semicircle equals the width of the rectangle)?

4. ([6 marks]) Find an equation of a line that is tangent to both of the curves $y = x^2$ and $y = x^2 - 2x + 2$ (at different points).

- 5. ([6 marks]) Let f(x) = x|x|.
 - (a) Using the definition of the derivative, show that f(x) is differentiable at x = 0.

(b) Find the second derivative of f(x). Explicitly state, with justification, the point(s) at which f''(x) does not exist, if any.

6. ([8 marks]) Use the definition of limit to prove that $\lim_{x\to 3} x^2 = 9$.

7. ([10 marks]) Let $f(x) = \sqrt{x}$.

(a) Find the third-order Taylor polynomial for f around x = 1.

(b) Evaluate

$$\lim_{t \to 0} \frac{f(1+t) - \cos(t/2) - \sin(t/2)}{t^3}.$$