Marks

[12] **1.** [Short answer]

(a) [3] Let $A = \begin{bmatrix} x & -y \\ y & x \end{bmatrix}$ and $B = \begin{bmatrix} s & -t \\ t & s \end{bmatrix}$. Is it true that $AB = BA$ for all choices of x, y, s and t?

(b) [3] Write down a 2×2 matrix with real entries but with complex eigenvalues.

(c) [3] For which values of a does $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ have only one eigenvector (up to scalar multiples)?

(d) [3] If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ what is A^{100}?

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[12] **2.** For which values of a and b are the vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ a \\ b \end{bmatrix}$ linearly independent?

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[14] **3.** Let T be the triangle in three dimensional space with vertices located at $\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

(a) [7] What is the cosine of the angle at the vertex $\mathbf{p}$?

(b) [7] What is the area of the triangle. (Hint: it is half the area of the parallelogram spanned by two of its sides.)

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[12] **4.** Consider the quadratic function

$$f(x, y) = 2x^2 + y^2 + 2xy - 8x - 6y + 16$$

Find the minimum value of f and where it occurs.

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[10] **5.** Let T be the linear transformation from three dimensionsal space $\mathbb{R}^3$ to $\mathbb{R}^3$ with

$$T\left(\begin{bmatrix}1\\2\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\-1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

What is the matrix for T?

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[9] **6.** Find the determinant of each of the following matrices and decide whether they are invertible.

(a) [3]

$$\begin{bmatrix} 1 & 2 & 1 & -1 & 4 & 9 \\ 0 & 2 & 0 & 0 & 8 & 8 \\ 0 & 0 & 1 & 1 & 5 & -5 \\ 0 & 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

(b) [3] The matrix product

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 6 & 2 \end{bmatrix}$$

(c) [3] Any 3×3 matrix with eigenvalues 1, 2 and 3. Give a reason.

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[16] **7.** The matrix P given by

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \end{bmatrix}$$

contains the transition probabilities for a random walk on three sites. The eigenvalues of P are 1, 1/3 and 0.

(a) [8] Find the eigenvector of P corresponding to the eigenvalue 1.

(b) [8] If the initial probabilities are given by a vector $\mathbf{x}$ with positive entries that sum to 1, find the limiting probabilities $\lim_{n\to\infty} P^n\mathbf{x}$.

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[15] **8.** Solve the system of differential equations

$$\begin{aligned} x_1'(t) &= -x_1(t) + 2x_2(t) \\ x_2'(t) &= -2x_1(t) - x_2(t) \end{aligned}$$

with initial conditions $x_1(0) = 1$, $x_2(0) = 1$. Write your final answer in a form that does *not* involve complex numbers.

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The University of British Columbia

Sessional Examinations - April 2006

Mathematics 152

Linear Systems

Closed book examination Time: 2.5 hours

Print Name _____ **Signature** _____

Student Number _____ **Instructor's Name** _____

Section Number _____

Special Instructions:

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You may bring one letter-sized formula sheet.
For all questions except the first, you must show your work (i.e., intermediate steps) for full credit.

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