The University of British Columbia

Final Examination - December 2011

Mathematics 200

Closed book examination — Time: 2.5 hours

Last Name: ______________________ **First:** ______________________

Student Number: ______________________

Signature: ______________________

Special Instructions:

- Be sure that this examination has 12 pages. Write your name at the top of each page.
- No books, notes, or calculators are allowed.
- Include explanations and simplify answers to obtain full credit.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

• Each candidate must be prepared to produce, upon request, a UBCcard for identification.

• Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

• No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

• Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;

(b) speaking or communicating with other candidates; and

(c) purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

• Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

• Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		12
2		12
3		12
4		14
5		14
6		10
7		10
8		16
Total		100

1. Consider the function $f(x, y) = e^{-x^2+4y^2}$.

   (a) Draw a "contour map" of f, showing all types of level curves that occur.

   (b) Find the equation of the tangent plane to the graph $z = f(x, y)$ at the point where $(x, y) = (2, 1)$.

   (c) Find the tangent plane approximation to the value of $f(1.99, 1.01)$ using the tangent plane from part (b).

2. Suppose $z = f(x, y)$ has continuous second order partial derivatives, and $x = r\cos t$, $y = r\sin t$. Express the following partial derivatives in terms r, t, and partial derivatives of f.

   (a) $\dfrac{\partial z}{\partial t}$

   (b) $\dfrac{\partial^2 z}{\partial t^2}$

3. A bee is flying along the curve of intersection of the surfaces $3z + x^2 + y^2 = 2$ and $z = x^2 - y^2$ in the direction for which z is increasing. At time $t = 2$, the bee passes through the point $(1, 1, 0)$ at speed 6.

   (a) Find the velocity (vector) of the bee at time $t = 2$.

   (b) The temperature T at position (x, y, z) at time t is given by $T = xy - 3x + 2yt + z$. Find the rate of change of temperature experienced by the bee at time $t = 2$.

4. Find the radius of the largest sphere centred at the origin that can be inscribed inside (that is, enclosed inside) the ellipsoid

$$2(x+1)^2 + y^2 + 2(z-1)^2 = 8.$$

Extra space (if needed)

5. (a) Consider the iterated integral

$$\int_{-4}^{0}\int_{\sqrt{-y}}^{2} \cos(x^3)\, dx\, dy$$

i. Draw the region of integration

ii Evaluate the integral

(b) Evaluate the double integral

$$\iint_D y\sqrt{x^2+y^2}\, dA$$

over the region $D = \{\, (x, y) \mid x^2 + y^2 \leq 2 \text{ and } 0 \leq y \leq x \,\}$.

6. Let R be the triangle with vertices $(0,2)$, $(1,0)$, and $(2,0)$. Let R have density $\rho(x,y) = y^2$. Find $\bar{y}$, the y-coordinate of the center of mass of R. **You do not need to find** $\bar{x}$.

7. Evaluate the triple integral $\iiint_E x \, dV$, where E is the region in the first octant bounded by the parabolic cylinder $y = x^2$ and the planes $y + z = 1$, $x = 0$, and $z = 0$.

8. The body of a snowman is formed by the snowballs $x^2 + y^2 + z^2 = 12$ (this is its body) and $x^2 + y^2 + (z - 4)^2 = 4$ (this is its head).

   (a) Find the volume of the snowman by subtracting the intersection of the two snow balls from the sum of the volumes of the snow balls. [Recall that the volume of a sphere of radius r is $\frac{4\pi}{3}r^3$].

(b) We can also calculate the volume of the snowman as a sum of the following triple integrals:

1.
$$\int_0^{\frac{2\pi}{3}} \int_0^{2\pi} \int_0^{2} \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi;$$

2.
$$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_{\sqrt{3}r}^{4-\frac{r}{\sqrt{3}}} r\, dz\, dr\, d\theta;$$

3.
$$\int_{\frac{\pi}{6}}^{\pi} \int_0^{2\pi} \int_0^{2\sqrt{3}} \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi.$$

Circle the right answer from the underlined choices and fill in the blanks in the following descriptions of the region of integration for each integral. [Note: We have translated the axes in order to write down some of the integrals above. The equations you specify should be those *before* the translation is performed.]

i. The region of integration in (1) is a part of the snowman's body / head / body and head.

It is the solid enclosed by the sphere / cone defined by the equation ________________________

and the sphere / cone defined by the equation ________________________.

ii. The region of integration in (2) is a part of the snowman's body / head / body and head.

It is the solid enclosed by the sphere / cone defined by the equation ________________________

and the sphere / cone defined by the equation ________________________.

iii. The region of integration in (3) is a part of the snowman's body / head / body and head.

It is the solid enclosed by the sphere / cone defined by the equation ________________________

and the sphere / cone defined by the equation ________________________.