The University of British Columbia

Final Examination - December 6, 2014

Mathematics 200

Closed book examination Time: 2 hours 30 minutes

Last Name: ______________ **First:** ______________

Student Number: ______________

Section (check one): ☐ 101 (MWF 9-10, Peterson) ☐ 102 (MWF 11-12, Fraser)
. ☐ 103 (MWF 11-12, Nguyen) ☐ 104 (MWF 1-2, Liu)
. ☐ 105 (TuTh 9:30-11, Roe) ☐ 107 (TuTh 3:30-5, Roe)

Special Instructions:

- Be sure that this examination has 13 pages. Write your name on top of each page.
- No books, notes, calculators, or any other aids are allowed.

1		7
2		10
3		10
4		8
5		14
6		14
7		14
8		14
9		9
Total		100

1. Suppose that $f(x, y, z)$ is a function of three variables and let $\mathbf{u} = \frac{1}{\sqrt{6}}\langle 1, 1, 2\rangle$ and $\mathbf{v} = \frac{1}{\sqrt{3}}\langle 1, -1, -1\rangle$ and $\mathbf{w} = \frac{1}{\sqrt{3}}\langle 1, 1, 1\rangle$. Suppose that at a point (a, b, c),

$$\begin{aligned} D_{\mathbf{u}}f &= 0 \\ D_{\mathbf{v}}f &= 0 \\ D_{\mathbf{w}}f &= 4. \end{aligned}$$

Find ∇f at (a, b, c).

2. Let $f(u, v)$ be a differentiable function of two variables, and let z be a differentiable function of x and y defined implicitly by $f(xz, yz) = 0$. Show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -z.$$

3. Let $z = f(x, y)$ be given implicitly by

$$e^z + yz = x + y.$$

(a) Find the differential dz.

(b) Use linear approximation at the point $(1, 0)$ to approximate $f(0.99, 0.01)$.

4. Let S be the surface $z = x^2 + 2y^2 + 2y - 1$. Find all points $P(x_0, y_0, z_0)$ on S with $x_0 \neq 0$ such that the normal line at P contains the origin $(0, 0, 0)$.

5. Let $f(x, y) = xy(x + y - 3)$.

   (a) Find all critical points of f, and classify each one as a local maximum, a local minimum, or saddle point.

(b) Find the location and value of the absolute maximum and minimum of f on the triangular region $x \geq 0$, $y \geq 0$, $x + y \leq 8$.

6. In the xy-plane, the disk $x^2 + y^2 \leq 2x$ is cut into 2 pieces by the line $y = x$. Let D be the larger piece.

   (a) Sketch D including an accurate description of the center and radius of the given disk. Then describe D in polar coordinates (r, θ).

   (b) Find the volume of the solid below $z = \sqrt{x^2 + y^2}$ and above D.

7. The density of hydrogen gas in a region of space is given by the formula

$$\rho(x, y, z) = \frac{z + 2x^2}{1 + x^2 + y^2}.$$

(a) At $(1, 0, -1)$, in which direction is the density of hydrogen increasing most rapidly?

(b) You are in a spacecraft at the origin. Suppose the spacecraft flies in the direction of $\langle 0, 0, 1\rangle$. It has a disc of radius 1, centred on the spacecraft and deployed perpendicular to the direction of travel, to catch hydrogen. How much hydrogen has been collected by the time that the spacecraft has traveled a distance 2? [You may use the fact that $\int_0^{2\pi} \cos^2\theta\, d\theta = \pi$.]

8. Consider the iterated integral

$$I = \int_{-a}^{0} \int_{-\sqrt{a^2-x^2}}^{0} \int_{0}^{\sqrt{a^2-x^2-y^2}} (x^2+y^2+z^2)^{2014}\, dz\, dy\, dx$$

where a is a positive constant.

(a) Write I as an iterated integral in cylindrical coordinates.

(b) Write I as an iterated integral in spherical coordinates.

(c) Evaluate I using whatever method you prefer.

9. Let E be the region bounded by $z = 2x$, $z = y^2$, and $x = 3$. The triple integral $\iiint_E f(x, y, z)\, dV$ can be expressed as an iterated integral in the following three orders of integration. Fill in the limits of integration in each case. No explanation required.

$$\int_{y=}^{y=} \int_{x=}^{x=} \int_{z=}^{z=} f(x, y, z)\; dz\, dx\, dy$$

$$\int_{y=}^{y=} \int_{z=}^{z=} \int_{x=}^{x=} f(x, y, z)\; dx\, dz\, dy$$

$$\int_{z=}^{z=} \int_{x=}^{x=} \int_{y=}^{y=} f(x, y, z)\; dy\, dx\, dz$$