Marks

[12] **1.** A bug is flying through space so that its coordinates at time t are $\mathbf{x}(t) = (t^2 + t, t^2 - t, t^3)$.

(a) (6 marks) Find the bug's velocity and acceleration for all t, and the curvature of its trajectory at time $t = 0$.

(b) (6 marks) Find all values of t at which the osculating plane is perpendicular to the xz-plane.

Continued on page 3

[10] **2.** Evaluate the integral $\int\int_D \sqrt{\dfrac{x+y}{x-2y}}\,dA$, if D is the region in $\mathbf{R}^2$ enclosed by the lines $y = x/2$, $y = 0$, and $x + y = 1$.

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[16] **3.** The following integral is given in cylindrical coordinates:

$$\int_0^{2\pi} \int_0^1 \int_{r\sqrt{3}}^{\sqrt{4-r^2}} r dz dr d\theta.$$

Sketch the region of integration. Convert the integral to equivalent iterated integrals in (a) Cartesian coordinates, (b) spherical coordinates. Evaluate the easiest of the three integrals.

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[8] **4.** Evaluate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$, if $\mathbf{F} = \sin y\mathbf{i} + (x\cos y - \cos z)\mathbf{j} + y\sin z\mathbf{k}$ and $\mathbf{x}$ is the parametrized curve $(\frac{\pi}{2}\sin\frac{\pi t}{2}, \pi t^2, \pi t^3)$, $0 \le t \le 1$. (Hint: this can be done without any complicated calculations.)

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[14] **5.**

(a) (6 marks) Evaluate $\int\int_{S_1} \mathbf{F} \cdot d\mathbf{S}$, if $\mathbf{F} = e^{x+y}\mathbf{i} - e^{x+y}\mathbf{j} + 2z\mathbf{k}$ and S_1 is the disc $x^2 + y^2 \leq 9$, $z = 3$, oriented so that the normal vector points upward.

(b) (8 marks) Evaluate $\int\int_{S_2} \mathbf{F} \cdot d\mathbf{S}$, if $\mathbf{F}$ is as in (a) and S_2 is the part of the sphere $x^2 + y^2 + (z-3)^2 = 9$ that lies above the plane $z = 3$, oriented so that the normal vector points upward. (Hint: use the Divergence Theorem.)

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[16] **6.** Let $\mathbf{F} = (x^2 + 3y^2z - 3z)\mathbf{i} + 3x^2z\mathbf{j} + (3y - x^3 - y^3)\mathbf{k}$.

(a) (8 marks) Prove that $(\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 0$ everywhere on S, if S is the cylinder $x^2 + y^2 = 1$ (with the normal vector pointing outward).

(b) (8 marks) Find $\int_C \mathbf{F} \cdot d\mathbf{s}$, if C is the oriented curve $(\cos t, \sin t, \sin t)$, $0 \le t \le \pi$. (Hint: using Stokes's theorem and part (a), reduce the problem to computing a simpler line integral.)

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[18] **7.** Let $\mathbf{X}$ be the parametrized surface $\mathbf{X}(s,t) = (st, s+t, s-t)$, $s^2 + t^2 \leq 1$.

(a) (6 marks) Find the surface area of $\mathbf{X}$.

(b) (6 marks) Find $\int\int_{\mathbf{X}} \mathbf{F} \cdot d\mathbf{S}$, if $\mathbf{F} = (y+z)^2\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(continued on next page)

(c) (6 marks) Find $\int_{\mathbf{X}} \omega$, if $\omega = (y - z)dx \wedge dz + xdz \wedge dy$.

[6] **8.** Decide whether the following regions are simply connected. (3 marks for each).

(a) The complement of the line segment from $(-1, 0)$ to $(1, 0)$ in $\mathbf{R}^2$.

(b) $\{(x, y, z) \in \mathbf{R}^3 : \ 1 \leq x^2 + y^2 + z^2 \leq 9\}$

The End

Be sure that this examination has 10 pages including this cover

The University of British Columbia

Sessional Examinations - April 2006

Mathematics 227

Advanced Calculus II

Closed book examination | Time: 2.5 hours

Print Name ____________________ **Signature** ____________________

Student Number ____________________ **Instructor's Name** ____________________

Section Number ____________________

Special Instructions:

No calculators, notes, or books of any kind are allowed.
Show all calculations for your solutions. If you need more space than is provided, use the back of the previous page.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.
2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		12
2		10
3		16
4		8
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6		16
7		18
8		6
Total		100