

Final 253

You have 150 min to solve 7 problems. Please note

- Write your name/student id clearly
- Write only on one side of the paper in the space design for it. Use the last few pages of the exam for calculations.

Good luck!

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

1. Find the limit or show that it does not exist

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$$



2. (a) Compute the gradient of the function

$$f(x, y) = \int_x^y \cos(t^2) dt$$

- (b) Use the gradient to approximate  $f(0, 0.1)$ .



3. If  $z = f(x, y)$  where  $x = s + t$  and  $y = s - t$  show that

(a)

$$(z_x)^2 - (z_y)^2 = (z_s)(z_t)$$

(b)

$$4(z_x)(z_y) = (z_s)^2 - (z_t)^2$$



4. Find the direction in which the directional derivative of the function

$$f(x, y) = ye^{-xy}$$

at point  $(0, 2)$  has the value 1.





5. a Find the critical points of the function

$$f(x, y) = -(x^2 - 1)^2 - (x^2y - x - 1)^2$$

- b Explore the points and check if they are minima/maxima or saddle points  
c In 3 lines or less, explain why the result you obtain does not have an equivalent when considering a function of a single variable.



6. Evaluate the integral

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1} dy dx$$



7. Evaluate the integral

$$I = \iint_D \frac{x - 2y}{3x - y} dA$$

where  $D$  is the domain enclosed by the lines

$$x - 2y = 0, \quad x - 2y = 4, \quad 3x - y = 1 \quad \text{and} \quad 3x - y = 8$$



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