THE UNIVERSITY OF BRITISH COLUMBIA
Sessional Examinations. April 2005

MATHEMATICS 257
Partial Differential Equations
and MATHEMATICS 316
Elementary Differential Equations II

Closed book examination **Time: 2 1/2 hours**

1) Calculators are not allowed in this examination.
2) A standard size (both sides) sheet of notes, is allowed in this examination.

I-[15] 1) Show that $x = 0$ is a rfegular singular point of the differential equation $x^2y'' + xy' - x^2y = 0$.

2) Find two linearly independent solutions of the differential equation near $x = 0$. What is the radius of convergence of the two power series solutions?

II-[15] Let

$$f(x) = \begin{cases} \pi & \text{if } -\pi \le x < 0 \\ \pi - x & \text{if } 0 \le x < \pi \end{cases}$$

with $f(x + 2\pi) = f(x)$.
1) Find the Fourier series corresponding to the given function $f(x)$.

2) Sketch the graph of the fnction to which the series converges over two periods.

III-[25] Solve the damped wave equation

$$\begin{aligned} u_{tt} + 2u_t &= u_{xx} - \sin x \ , \ 0 < x < \pi,\ 0 < t \\ u(0,t) = 0 \quad &, \quad u_x(\pi, t) = 2 \\ u(x,0) = f(x) \quad &, \quad u_t(x,0) = 0 \ , \ 0 < x < \pi. \end{aligned}$$

Hint Write $u(x,t) = v(x) + w(x,t)$.

IV-[25] Solve the boundary value problem

$$\begin{aligned} u_{xx} + u_{yy} + 2u_y + u &= 0, \;\; 0 < x < 1, \; 0 < y < 1 \\ u(0, y) = 0 &= u(1, y) \;\; , 0 < y < 1 \\ u(x, 0) = 3\sin(5\pi x) \quad &, \quad u_y(x, 1) = 2\sin(3\pi x) \end{aligned}$$

V-[20] Solve the initial boundary value problem

$$\begin{aligned} u_t - u_{xx} &= t\sin(5\pi x)\,, \;\; 0 < x < 1, \; 0 < t \\ u(0, t) = 0 &= u(1, t) \\ u(x, 0) &= x(1 - x), \; 0 < x < 1. \end{aligned}$$