The University of British Columbia

Final Examination - December 5, 2008

Mathematics 257/316

All Sections

Closed book examination | Time: 2.5 hours

Last Name: ____________________ , **First:** ______________ **Signature** ____________________
(USE CAPITALS)

Student Number ________________ **Instructor's Name & Section** ____________________

Special Instructions:

- Students are not allowed to bring any notes into the exam.
- No calculators are allowed.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		20
2		25
3		15
4		20
5		20
Total		100

[20] **1**. Consider the differential equation

$$4x^2y'' + xy' - (1 + 3x)y = 0 \tag{1}$$

(a) Classify the points $0 \leq x < \infty$ as ordinary points, regular singular points, or irregular singular points.

(b) Find two values of r such that there are solutions of the form $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$.

(c) Use the series expansion in (b) to determine two independent solutions of (1). You only need to calculate the first three non-zero terms in each case.

(d) Determine the radius of convergence of the series in (c).

(Question 1 Continued)

[25] **2**. Consider the following initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx}, \quad 0 < x < 1, \quad t > 0 \\ u_x(0,t) &= 1 \quad \text{and} \quad u(1,t) = 0 \\ u(x,0) &= \cos(3\pi x/2), \quad 0 < x < 1 \end{aligned} \tag{2}$$

(a) Determine a steady state solution to the boundary value problem. [**5 marks**]

(b) Use this steady state solution to determine the solution to the boundary value problem (2) by separation of variables. [**10 marks**]

(c) Briefly describe how you would use the method of finite differences to obtain an approximate solution this boundary value problem. Use the notation $u_n^k \simeq u(x_n, t_k)$ to represent the nodal values on the finite difference mesh. Explain how you propose to approximate the boundary conditions. [**10 marks**]

Hint: It might be useful to know that $\int_0^1 (1-x)\cos\left(\left(\frac{2n+1}{2}\right)\pi x\right) dx = \frac{4}{\pi^2(2n+1)^2}$

(Question 2 Continued)

(Question 2 Continued)

[15] **3**. The displacement $u(x,t)$ of a string of length 1 subject to viscous damping satisfies the damped wave equation

$$u_{tt} + 2u_t = u_{xx}$$

The string is set in motion from its initial displacement $u = f(x)$ from rest while both the ends of the string are held fixed. Use separation of variables to solve for the displacement of the string as a function of time by solving the following boundary value problem:

$$\begin{aligned} u_{tt} + 2u_t &= u_{xx} \qquad 0 < x < 1,\ t > 0 \\ u(0,t) &= 0, \qquad u(1,t) = 0 \\ u(x,0) &= f(x), \qquad u_t(x,0) = 0 \end{aligned}$$

Since $f(x)$ is not specified, you may leave the expressions for the Fourier coefficients as integrals.

(Question 3 Continued)

[20] **4**. Use separation of variables to solve the following mixed boundary value problem for the semi-circular region:

$$
\begin{aligned}
u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} &= 0, \quad 0 < r < a, \quad 0 < \theta < \pi \\
u(r,0) &= 0 \quad \text{and} \quad \frac{\partial u}{\partial \theta}(r,\pi) = 0 \\
u(r,\theta) &< \infty \text{ as } r \to 0 \quad \text{and} \quad u(a,\theta) = 1
\end{aligned}
$$

(Question 4 Continued)

[20] **5**. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + e^{-2t}\sin(5\pi x) + (1-x),\ 0 < x < 1,\ t > 0 \\ u(0,t) &= t \quad \text{and} \quad u(1,t) = 0 \\ u(x,0) &= \sin(\pi x) \end{aligned}$$

(Question 5 Continued)

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