[12] **1.** (a) Find the *scalar* projection of the vector $\langle 1, \pi, 2\pi \rangle$ along the vector $\langle 2, 6, 3 \rangle$.

(b) Find the *vector* projection of $\langle 1, \pi, 2\pi \rangle$ along the vector $\langle 2, 6, 3 \rangle$.

(c) Find the distance of the point $(2, \pi + 14, 2\pi - 27)$ from the plane given by the equation $2(x - 1) + 6(y - 14) + 3(z + 27) = 0$.

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[12] **2.** In an experiment, the light intensity is an unknown function $I(x, y, z)$ (it depends on the point in space). Its gradient at the point $(1, 2, 3)$ is given:

$$\nabla I|_{(1,2,3)} = \langle 0.1, -0.2, 0.5\rangle$$

(we ignore the units).

(a) Find the directional derivative $D_u I$ in the direction of the vector $\langle 1, 4, 12\rangle$.

(b) A small light sensor is moving through this space, with the position function

$$\mathbf{r}(t) = \langle t, 2t^2, 3t^4\rangle.$$

Find the velocity of the sensor at the time $t = 1$.

(c) Find $\frac{dI}{dt}|_{t=1}$ – the rate of change of intensity that the sensor (with the position function from part (b)) registers at the time $t = 1$.

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[10] **3.** Find the absolute minimum of the function

$$f(x, y) = 2x^2 + y^2 + xy - 5x - 3y + 4$$

on the closed triangle bounded by the x-axis, the line $x = 2$, and the line $y = 2x$. (For full credit, it is sufficient to list *all* the points where you have to evaluate the function to find the minimum; you do not have to evaluate the function at all these points).

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[10] **4.** The double integral of some function over a domain D is represented by the iterated integral as follows:

$$\iint_D f(x,y)\,dA = \int_0^1 \int_{\sqrt{x}}^{2\sqrt{x}} f(x,y)\,dy\,dx.$$

(a) Sketch the domain D.

(b) Change the order of integration, so that in the result, integration with respect to y is on the outside.

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[8] **5.** A wire that occupies the segment in space between the points $(0, 1, 1)$ and $(2, 1, 4)$ has the density given by the formula

$$\rho(x, y, z) = ye^x + z,$$

(we ignore the units). Find the total mass of the wire.

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[15] **6.** The vector field $\mathbf{F}$ is defined on the plane by the formula

$$\mathbf{F}(x, y) = \langle y^2 \cos(xy^2),\ 2xy \cos(xy^2) + 3y^2 \rangle.$$

(a) Is $\mathbf{F}$ conservative? Explain why or why not.

(b) Find a potential function for $\mathbf{F}$ if it exists, or explain why it does not exist.

(c) Find the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ in any way you like, where C is the curve parametrized by the function $\mathbf{r}(t) = \langle t^2, t^3 \rangle$, $0 \leq t \leq 1$. (Here $\mathbf{F}$ is the same vector field as in parts (a), and (b)).

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[15] **7.** The vector field $\mathbf{F}$ is defined on the whole space by the formula

$$\mathbf{F}(x, y, z) = \langle y^3 + e^{4x^2}, -x^3 - \sin(y), 3z^4 \rangle.$$

(a) Find **curl** ($\mathbf{F}$).

(b) Evaluate (by direct computation) the flux integral

$$\iint_S \mathbf{curl}\ (\mathbf{F}) \cdot d\mathbf{S},$$

where S is the surface defined by $x^2 + y^2 \leq 1$, $z = 0$, (the unit disc in the xy-plane), oriented upward.

(This question is continued on the next page).

(c) Now let $\mathbf{F}$ be the same vector field as in parts (a) and (b), and let M be the hemisphere $x^2 + y^2 + z^2 = 1$, $z \leq 0$, oriented downward. Evaluate

$$\iint_M \mathbf{curl}\ (\mathbf{F}) \cdot d\mathbf{S}$$

in any way you like.

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[18] **8.** (a) Let E be the solid inside the sphere of radius 2 centred at the origin, and above the cone defined by the equation $z^2 = \frac{1}{3}(x^2 + y^2)$. Using spherical coordiantes, evaluate

$$\iiint_E z\, dV.$$

This problem continues on the next page.

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(b) Let $\mathbf{F}$ be the vector field defined by the formula

$$\mathbf{F}(x, y, z) = \langle xz + y^2,\ x^2z^3,\ z^2 - y\rangle.$$

Compute **div** $(\mathbf{F})$.

(c) Let S be the closed surface that encloses the solid E from part (a) (so that S consists of the part of the cone that lies inside the sphere, and the spherical cap), oriented outward. Compute, in any way you like, the flux integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

The End

Be sure that this examination has 11 pages including this cover

The University of British Columbia

Sessional Examinations - December 2010

Mathematics 263

Multivariable and Vector Calculus

Closed book examination

Time: 2.5 hours

Surname(s): ____________________ **Given Name(s):** ____________________

Student Number: ____________________ **Instructor's Name:** ____________________

Signature: ____________________ **Section Number:** ____________________

Special Instructions:

No books, notes, cell-phones or calculators are allowed, except for one letter-size formula sheet (you can use both sides). You must show all your work (i.e., intermediate steps) for full credit. If you need more space, use the back of the previous page, or ask for a booklet. You must turn in all the booklets you use.

Rules governing examinations

1		12
2		12
3		10
4		10
5		8
6		15
7		15
8		18
Total		100