University of British Columbia
Math 301, Section 201

Final exam

Date: April 18, 2012
Time: 8:30 - 11:00am

Name (print):
Student ID Number:
Signature:

Instructor: Richard Froese

Instructions:

1. No notes, books or calculators are allowed. A summary sheet with properties of Fourier and Laplace transforms is provided.
2. Read the questions carefully and make sure you provide all the information that is asked for in the question.
3. Show all your work. Answers without any explanation or without the correct accompanying work could receive no credit, even if they are correct.
4. Answer the questions in the space provided. Continue on the back of the page if necessary.

Question	Mark	Maximum
1		14
2		14
3		15
4		14
5		15
6		14
7		14
Total		100

[14] 1. Evaluate

$$I = \int_0^\infty \frac{\cos(\pi x)}{1 - 4x^2}\,dx,$$

explaining all your steps.

[14] 2. Evaluate

$$I = \int_0^\infty \frac{x^{\alpha-1}}{1+x} dx, \quad 0 < \alpha < 1.$$

explaining all your steps.

3. Let $g(z)$ be the branch of $(z(z-1))^{1/2}$ defined by $g(z) = \exp\left((\mathrm{Log}(z) + \mathrm{Log}(z-1))/2\right)$, where Log denotes the principal branch of the logarithm.

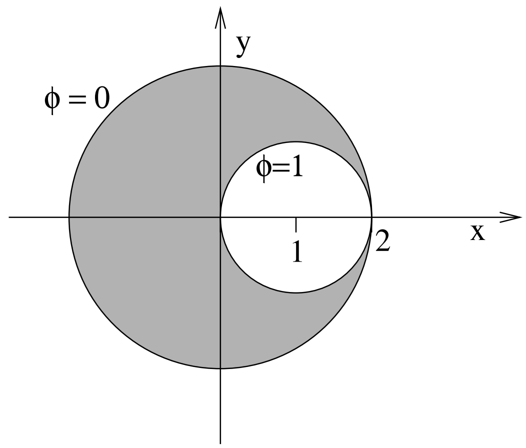
[4] (a) Compute $\lim_{z \to -1} g(z)$ when z approaches -1 from the upper half plane, and from the lower half plane. Is $g(z)$ continuous at -1?

[3] (b) Where are the branch cuts for $g(z)$?

[4] (c) Show how to construct $g(z)$ using the ‘range of angles’ method.

[4] (d) What is the value of $g(i)$ and $g(-i)$?

[14] 4. Solve Laplaces equation $\Delta\phi = 0$ in the region between the circles $\{z : |z| = 2\}$ and $\{z : |z-1| = 1\}$ with boundary condition $\phi = 1$ on the inner circle and $\phi = 0$ on the outer circle.



5. Consider the complex velocity potentials for ideal inviscid flow given by $\Omega_1(z) = iV_0(z - 1/z)$ and $\Omega_2(z) = i\gamma \log(z)$. Here $V_0 > 0$ and $\gamma > 0$.

[4] (a) Show that $\Omega_1(z)$ describes a flow around the unit circle $\{z : |z| = 1\}$.

[4] (b) What are the streamlines for the flow described by $\Omega_2(z)$? Plot several of these, and indicate with arrows the direction of the flow.

[3]

(c) Explain why $\Omega(z) = \Omega_1(z)+\Omega_2(z)$ also describes a flow around the unit circle $\{z : |z| = 1\}$. What is the asympotic velocity of this flow as $|z| \to \infty$?

[4]

(d) Where are the stagnation point for the flow described by $\Omega(z)$? For what values of $\gamma > 0$ are the stagnation points on the boundary of the the unit circle?

[14] 6. Solve

$$u_t(x,t) = u_{xx}(x,t) + u(x,t). \quad -\infty < x < \infty, \quad t > 0$$
$$u(x,0) = e^{-x^2}$$

[7]

7. (a) What is the Laplace transform $Y(s)$ of the solution $y(t)$ to the initial value problem

$$y'''(t) - 6y''(t) + 11y'(t) - 6y(t) = e^{-2t}, \quad y(0) = y'(0) = 0, \quad y''(0) = 1?$$

[7]

(b) Use the Nyquist criterion to determine whether the solution $y(t)$ is bounded as t tends to infinity.