The University of British Columbia

Final Examination - December 10, 2009

Mathematics 308

Section 101

Closed book examination Time: 2.5 hours

Last Name: ____________________ , **First:** ______________ **Signature** ____________________

Student Number ____________________

Special Instructions:

- No books, notes or calculators are allowed.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		12
3		14
4		16
5		14
6		12
7		12
8		10
Total		100

[10] **1**. In Fig. 1, $AB = AC$, both ΔABD and ΔACF are equilateral triangles. If G is the intersection point of AB and CF, and H is the intersection point of AC and BD, prove that $AG = AH$.

[12] **2**. In Fig. 2, ABC is a right triangle with $\angle ACB = 90°$, $ACDE$, $BCNM$ and $ABHF$ are three squares. If $GF = BC$ and $\angle GFH = \angle ABC$, prove that the quadrilateral $ABME$ and the quadrilateral $AFGC$ are congruent by addition, i.e., $ABME \simeq AFGC(+)$.

[14] **3**. Assume that $\Omega_\ell \tau_v = \tau_{kv} \Omega_\ell$, where Ω_ℓ is the reflection in a line ℓ, τ_v is the translation determined by a non-zero vector v and k is a real constant. Show that $v // \ell$ or $v \perp \ell$.

[16] **4.** Let $T : \mathbb{E}^2 \to \mathbb{E}^2$ be a function defined by

$$T(x, y) = (y + 1, x + 1) \qquad \text{for } (x, y) \in \mathbb{E}^2.$$

(i) Prove that T is an isometry?

(ii) Find the fixed line(s) of T.

(iii) Is T a glide reflection? If your answer is YES, find a vector v and a line ℓ such that $T = T_v\Omega_\ell$, where Ω_ℓ is the reflection in the line ℓ and τ_v is the translation determined by the vector v. If your answer is NO, give your reason.

[14] **5**. Let $A(1,2)$, $B(0,0)$, $C(4,0)$ and $H(2,2)$ be four points in $\mathbb{E}^2$. Assume that T is central dilatation such that $T(A) = B$ and $T(H) = C$.

(i) If $M(a,b)$ is the center of T, find a and b.

(ii) Find $T(R)$, where R is the point $\left(\sqrt{2}, \sqrt{3}\right)$.

[12] **6**. In Fig. 3, $M_1M_2M_3H_1H_2H_3N_1N_2N_3$ is the nine-point circle of $\Delta A_1A_2A_3$ and H is the orthocenter of $\Delta A_1A_2A_3$.

(i) Which point in Fig. 3 is the orthocenter of ΔA_1A_2H?

(ii) If the area of the circle which passes through A_1, A_2 and A_3 has the area π, find the area of the nine-point circle of A_1A_2H.

[12] **7**. Let $A(1,0)$, $B(5,0)$ and $C(2,0)$ be three points in $\mathbb{E}^2$. Find a point $D(x,0)$ such that the cross ratio (AB,CD) is equal to $-\frac{1}{2}$.

[10] **8**. In a triangle ABC, prolong BA and CA to get two parallelograms $AGHC$ and $ALKB$ (see Fig. 4). Show that if AD, BH and CK are concurrent and D is the midpoint of BC, then the two parallelograms $AGHC$ and $ALKB$ have the same areas.
(Hint: the area of the parallelogram $AGHC$ is equal to $AG \cdot AC \cdot \sin \angle GAC$.)