The University of British Columbia
Final Examination - December 7th, 2006

Mathematics 317, joint final
Instructors: Jim Bryan and Alexander Roitershtein

Closed book examination | Time: 2.5 hours

Name ______________________ **Signature** ______________________

Student Number ______________

Special Instructions:
- Be sure that this examination has 12 pages. Write your name on top of each page.
- No calculators or notes are permitted.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

• Each candidate should be prepared to produce her/his library/AMS card upon request.
• No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
• Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
• CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
(a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.
(b) Speaking or communicating with other candidates.
(c) Purposely exposing written papers to the view of other candidates.

1		12
2		13
3		12
4		13
5		12
6		13
7		12
8		13
Total		100

Problem 1. (12 points.)
Evaluate the integral

$$\int_C xy\,dx + yz\,dy + zx\,dz$$

around the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, oriented clockwise as seen from the point $(1, 1, 1)$.

Problem 2. (13 points.)
Let C be the curve in the xy plane from the point $(0,0)$ to the point $(5,5)$ consisting of the ten line segments consecutively connecting the points (0,0), (0,1), (1,1), (1,2), (2,2), (2,3), (3,3), (3,4), (4,4), (4,5), (5,5). Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where

$$\mathbf{F} = y\mathbf{i} + (2x - 10)\mathbf{j}.$$

Problem 3. (12 points.)
Let S be the surface given by the equation

$$x^2 + z^2 = \sin^2(y)$$

lying between the planes $y = 0$ and $y = \pi$. Evaluate the integral

$$\iint_S \sqrt{1 + \cos^2(y)}\ dS.$$

Problem 4. (13 points.)
Let S be the part of the sphere $x^2+y^2+z^2=4$ between the planes $z=1$ and $z=0$ oriented away from the origin. Let

$$\mathbf{F} = (e^y + xz)\,\mathbf{i} + (zy + \tan(x))\mathbf{j} + (z^2 - 1)\mathbf{k}.$$

Compute the flux integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

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Problem 5. (12 points.)
Let

$$\mathbf{r}(t) = \cos^3 t\ \mathbf{i} + \sin^3 t\ \mathbf{j} + \frac{3}{2}\sin t \cos t\ \mathbf{k}.$$

Reparameterize $\mathbf{r}(t)$ with respect to arclength measured from the point $t = 0$ in the direction of increasing t.

Problem 6. (13 points.)
Let

$$\mathbf{r}(t) = t^2\,\mathbf{i} + 2t\,\mathbf{j} + \ln t\,\mathbf{k}.$$

Compute the unit tangent and unit normal vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$. Compute the curvature $\kappa(t)$. **Simplify whenever possible!**

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Problem 7. (12 points.)
Show that the following line integral is independent of path and evaluate the integral.

$$\int_C (ye^x + \sin y)dx + (e^x + \sin y + x\cos y)dy,$$

where C is any path from $(1, 0)$ to $(0, \pi/2)$.

Problem 8. (13 points.)
Let

$$\mathbf{F} = \frac{-z}{x^2+z^2}\,\mathbf{i} + y\,\mathbf{j} + \frac{x}{x^2+z^2}\,\mathbf{k}.$$

1. Determine the domain of $\mathbf{F}$.
2. Determine the curl of $\mathbf{F}$. Simplify if possible.
3. Determine the divergence of $\mathbf{F}$. Simplify if possible.
4. Is $\mathbf{F}$ conservative? Give a reason for your answer.

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