PUTNAM PRACTICE SET 11

PROF. DRAGOS GHIOCA

Problem 1. Find the sum of the series

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2n}{3^m(3^mn+3^nm)}.$$

Problem 2. Prove that there exists a positive constant C such that for any polynomial $P\in\mathbb{R}[x]$ of degree less than 2020, we have that

$$P(0)\leq C\cdot\int_{-1}^{1}|P(x)|\mathrm{dx}.$$

Problem 3. The sequence $\{a_n\}$ satisfies

$$a_1=1;\ a_2=2;\ a_3=24 \text{ and for } n\geq 4:$$
$$a_n=\frac{6a_{n-1}^2a_{n-3}-8a_{n-1}a_{n-2}^2}{a_{n-2}a_{n-3}}.$$

Prove that for each positive integer n, we have that a_n is an integer multiple of n.

Problem 4. Let $P\in\mathbb{C}[x]$ be a polynomial of degree n such that $P(x)=Q(x)\cdot P''(x)$, where $Q(x)$ is a quadratic polynomial and P'' is the double derivative of P. Show that if $P(x)$ has at least two distinct roots, then it must have n distinct roots.